CPS-oriented Modeling and Control of Traffic Signals Using Adaptive Back Pressure

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***Abstract*—Modeling and design of automotive systems from a cyber-physical system (CPS) perspective have lately attracted extensive attention. As the trend towards automated driving and connectivity accelerates, strong interactions between vehicles and the infrastructure are expected. This requires modeling and control of the traffic network in a similarly formal manner. Modeling of such networks involves a tradeoff between expressivity of the appropriate features and tractability of the control problem. Back-pressure control of traffic signals is gaining ground due to its decentralized implementation, low computational complexity, and no requirements on prior traffic information. It guarantees maximum stability under idealistic assumptions. However, when deployed in real traffic intersections, the existing back-pressure control algorithms may result in poor junction utilization due to (i) fixed-length control phases; (ii) stability as the only objective; and (iii) obliviousness to finite road capacities and empty roads. In this paper, we propose a CPS-oriented model of traffic intersections and control of traffic signals, aiming to address the utilization issue of the back-pressure algorithms. We consider a more realistic model with transition phases and dedicated turning lanes, the latter influencing computation of the pressure and subsequently the utilization. The main technical contribution is an adaptive controller that enables varying-length control phases and considers both stability and utilization, while taking both cases of full roads and empty roads into account. We implement a mechanism to prevent frequent changes of control phases and thus limit the number of transition phases, which have negative impact on the junction utilization. Microscopic simulation results with SUMO on a 3×3 traffic network under various traffic patterns show that the proposed algorithm is at least about $13\,\%$ better in performance than the existing fixed-length back-pressure control algorithms reported in previous works. This is a significant improvement in the context of traffic signal control.**

I. INTRODUCTION

Following the trend towards automated driving and the development of communication networks, strong interactions between vehicles and the infrastructure are expected in the future intelligent transportation. While there has been a strong interest in modeling and design of automotive systems from a cyber-physical system (CPS) perspective, the traffic network also needs to be modeled and controlled in a similarly formal manner. Traffic signal control has a significant impact on the performance of the transportation network. In a traffic intersection, a set of compatible rights-of-way are signaled to vehicles and referred to as a control phase. There have recently been works on intelligent traffic signal control and behavior adaptation from traffic signal prediction [1], [2]. In particular, back-pressure traffic signal control is gaining ground [3], [4], [5]. It implements a state-feedback controller (cyber) based on real-time queue lengths (physical) at the intersection. A longer queue indicates a larger pressure. The general idea is to always select the control phase that makes the best efforts in reducing the pressure difference and balancing the queues. The main advantages of this algorithm are its decentralized implementation, low computational complexity, and no requirements on prior traffic information. It also guarantees maximum stability (in terms of bounded queue lengths) under idealistic assumptions that the control phase change is immediate and can be triggered at any time.

However, the existing back-pressure control algorithms that can be realistically applied in practice often lead to poor junction utilization due to three major reasons. (i) A control phase is activated for a pre-determined time slot based on the pressures exerted by the queues, taken at the beginning of the time slot. It does not react to the real-time evolvement during this fixed-length time slot and may not get the most vehicles served by the junction. (ii) When there is a conflict between stability and utilization, utilization is ignored. (iii) The conventional back-pressure control is oblivious to finite road capacities and empty roads. If an outgoing road of an intersection reaches its capacity (e.g., during heavy traffic), it will not be able to accommodate any new incoming vehicle, until the old vehicles get served by the neighbor intersection. When an incoming road is empty (e.g., during light traffic), the junction utilization will be low, as it takes some interval for the next vehicle to arrive at the intersection and get served.

Main Contributions: In this paper, we propose a CPS-oriented model of traffic intersections and control of traffic signals, to improve the utilization of the back-pressure algorithms. We study a more realistic model of the intersection with transition phases and dedicated turning lanes. The latter influences computation of the pressure and subsequently the utilization. An adaptive controller that enables varying-length control phases and considers both stability and utilization is reported. While frequent change of control phases tends to reflect the real-time queue lengths and thus improve the junction utilization, the increasing number of transition phases (with the amber light on) considerably lowers the junction utilization. Therefore, we implement a mechanism to limit the change of control phases. In addition, the proposed control algorithm is aware of the low utilization resulting from full outgoing and empty incoming roads. Simulation results with

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SUMO — a widely used microscopic traffic simulator — on a 3×3 traffic network under various traffic patterns show that the proposed algorithm is at least about $13\,\%$ better than the state of the art [4] (there have been works applying back-pressure more recently, but they are studying different issues, such as [5] on routing). This is a significant improvement in the context of traffic signal control.

The rest of this paper is organized as follows. Section II presents the system and control model. The proposed algorithm is explained in Section III. Properties of the algorithm are studied in Section IV. Simulations results are reported in Section V and Section VI makes concluding remarks.

II. SYSTEM MODELING

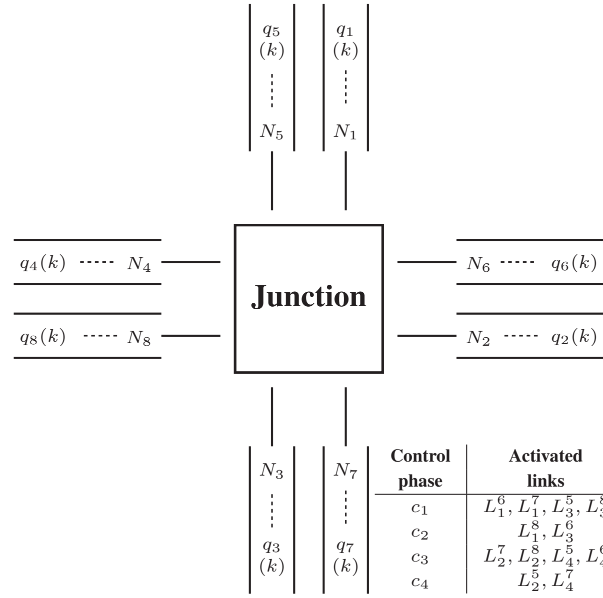
In this paper, we study transportation systems from a CPS-oriented perspective. Here, the physical process is the traffic flow across signalized intersections, and the cyber part mainly consists of a controller regulating the traffic flow. Accurate modeling of the physical process is the key to develop effective control algorithms. In this work, we apply the standard queuing network model [6] and extend it for more realistic consideration of the traffic network. On the other hand, phase-based control algorithms are investigated.

A. Signalized Intersection

According to the queuing network model, a signalized intersection can be represented as a directed graph. Each node represents a road participating in the traffic flow through the junction. The set of these roads is denoted as $N = \{N_i \mid i \in \mathcal{N}\}$, where $\mathcal{N} = \{1, 2, \ldots\}$ and $|\mathcal{N}|$ is the total number of roads at the intersection. The subset $NI = \{N_i \in N \mid i \in \mathcal{NI}\}$ comprises the roads from which vehicles enter the junction, i.e., incoming roads. The subset $NO = \{N_i \in N \mid i \in \mathcal{NO}\}$ comprises the roads from which vehicles leave the junction, i.e., outgoing roads. We have $\mathcal{N} = \mathcal{NI} \cup \mathcal{NO}$ and $\mathcal{NI} \cap \mathcal{NO} = \varnothing$. The nodes in the graphs are connected via directed links denoted as $L = \left\{ L_i^{i'} \mid i \in \mathcal{NI}, i' \in \mathcal{NO} \right\}$. If the traffic flow from N_i to $N_{i'}$ is legal, the link $L_i^{i'}$ is said to be feasible and can be activated. The controller managing the intersection is allowed to activate a set of compatible links without leading to conflicting traffic flows. As illustrated in Figure 1, the example intersection has eight nodes with four incoming roads $\{N_1, N_2, N_3, N_4\}$ and four outgoing roads $\{N_5, N_6, N_7, N_8\}$. There are twelve feasible links. For instance, activating the link L_1^6 enables vehicles on the road N_1 to turn left and enter the road N_6 via the junction.

In this work, we consider a discrete-time system and the state of the intersection is monitored at discrete instants of time, denoted by k ($k \in \mathbb{N}$), due to the working principles of the sensors. We assume that each road has dedicated turning lanes near a junction, according to which vehicles queue. For an incoming road, we are interested in the queue length towards each outgoing road. We denote $q_i^{i'}(k)$ as the number of vehicles queuing at $N_i \in NI$ going to $N_{i'} \in NO$ at time k. The total queue length at $N_i \in NI$ can be calculated as

$$q_i(k) = \sum_{i' \in \mathcal{NO}} q_i^{i'}(k). \tag{1}$$



Control phase	Activated links
c_1	$L_1^6, L_1^7, L_3^5, L_3^8$
c_2	L_1^8, L_3^6
c_3	$L_2^7, L_2^8, L_4^5, L_4^6$
c_4	L_2^5, L_4^7

Fig. 1. An example intersection with four incoming nodes and four outgoing nodes. Activated links corresponding to four control phases are tabularized.

For an outgoing road $N_{i'} \in NO$, we are only interested in the total queue length $q_{i'}(k)$, which is assumed to be known at time k. The capacity of N_i, as denoted by W_i, is the maximum number of vehicles that the road N_i can accommodate. When W_i is reached, no vehicles are able to enter N_i.

B. Arriving and Queuing Vehicles

We assume that the arrival of vehicles at each incoming road is an exogenous process, modeled by a discrete random variable $\mathcal{X}$, which has a Poisson distribution with the rate $\lambda > 0$ [6]. The number of vehicles arriving at the road $N_i \in NI$ from the time instant k to $k+1$ going to the road $N_{i'} \in NO$ is denoted as $A_i^{i'}(k, k+1)$. The queuing dynamics can be written as

$$q_i^{i'}(k+1) = q_i^{i'}(k) + A_i^{i'}(k, k+1) - S_i^{i'}(k, k+1), \tag{2}$$

where $S_i^{i'}(k, k+1)$ is the number of vehicles leaving N_i for $N_{i'}$ (served by the junction) in the period between k and $k+1$.

C. Control Phases and Service Dynamics

The set of feasible control phases at an intersection is denoted by $C = \{c_j\}$, and $c_j \subset L$. Corresponding to each phase c_j, a compatible subset of L is activated. For the example shown in Figure 1, there are four control phases in total, i.e., $C = \{c_1, c_2, c_3, c_4\}$. For instance, when c_2 is applied, the links L_1^8 and L_3^6 are activated, allowing vehicles queuing at N_1 and N_3 to make a right turn. It is noted that the transition phase (i.e., the period when the amber light is on to clear vehicles in the junction) is denoted as $c_0 = \varnothing$. During this phase, no links are activated.

Vehicles get served by the junction depending on the applied control phase. We assume that the full service rate for vehicles going from N_i to $N_{i'}$ is $\mu_i^{i'}$. The maximum number of vehicles that can be served during the period of Δt is then $\mu_i^{i'} \Delta t$. There are three factors determining if this maximum is reached. First, the link from N_i to $N_{i'}$ has to be activated by the applied control phase c_j, i.e., $L_i^{i'} \in c_j$. Second, the number of vehicles waiting to be served via the link $L_i^{i'}$ during Δt must be at least $\mu_i^{i'} \Delta t$. Third, the queue at $N_{i'}$ cannot exceed its capacity $W_{i'}$.

III. Utilization-Aware Adaptive Traffic Signal Control

A. Proposed Metrics and Definitions

The back-pressure control essentially implements a state-feedback control law. At every time instant, it selects the phase to apply based on the system state, i.e., the lengths of the queues at the intersection:

$$c(k) = \phi(Q(k)). \tag{3}$$

The control law is ϕ and $c(k) \in C$ is the selected control phase. The set of all queue lengths is $Q(k) = \{q_i \mid i \in \mathcal{NO}\} \cup \left\{q_i^{i'} \mid L_i^{i'} \in L\right\}$.

The function $b = f(q)$ is used to map the queue length to a pressure value. In this work, we consider the mapping function to be

$$b = f(q) = q. \tag{4}$$

In the original back-pressure traffic signal control algorithm, decisions are made based on a metric, which we will refer to as link gain. For the original case, this link gain at the time instant k is calculated as

$$g_o(L_i^{i'}, k) = max(0, (b_i(k) - b_{i'}(k))\mu_i^{i'}). \tag{5}$$

Note that the gain is always non-negative. When the gain is positive, it is given by the product of two terms: (i) the pressure difference between the incoming and outgoing road of a link; (ii) the maximum service rate of the link. The first term implies the degree of imbalance of the link, and the second term suggests how fast the pressure difference can be balanced. The link with a higher gain has the priority, and the phase with the highest overall gain (sum of constituent link gains) is activated. When all the gains are 0, no phase is activated.

In this work, we introduce a modified link gain as

$$g(L_i^{i'}, k) = (b_i^{i'}(k) - b_{i'}(k) + W^*)\mu_i^{i'}, \tag{6}$$

where W^* is given by

$$W^* = \max_{i' \in \mathcal{NO}} W_{i'}. \tag{7}$$

There are two differences. (i) We replace b_i with $b_i^{i'}$. That is, we consider that the pressure at the incoming road is only exerted by the queue using the link. This is more reasonable since vehicles not intending to use the link should not contribute to the link gain. Otherwise, a high link gain may get the link activated, but lead to a poor utilization of the junction and even deadlocks. (ii) We allow negative pressure differences. When all the pressure differences are negative, there could still be vehicles to be served by the junction. We add a constant W^* to make the first term always positive. Therefore, the link gain gets higher with a larger pressure difference or maximum service rate. Although a different metric is defined, the essence of the back-pressure algorithm is kept. In general, we still try to balance the pressures exerted by the queues and thus stabilize the system, while taking the junction utilization into account. This will be further explained later in detail.

In principle, as discussed above, the link with a higher gain should be prioritized in getting activated. However, there are two special scenarios to consider. First, the outgoing node $N_{i'} \in NO$ reaches its capacity, i.e., $q_{i'} = W_{i'}$. This can happen during heavy traffic. No vehicles can enter $N_{i'}$ until the queuing vehicles start getting served by the neighbor intersection. The utilization will be very low in this case and it is not wise to activate the link $L_i^{i'}$, no matter what gain value it has. Second, there are no vehicles queuing at the incoming node N_i going to $N_{i'}$, i.e., $q_i^{i'} = 0$. This can occur during light traffic. If $L_i^{i'}$ is activated in this case, only newly arriving vehicles will be served, resulting in low junction utilization. Taking the above scenarios into account, we update (6) with (8).

$$g(L_i^{i'}, k) = \begin{cases} \beta & : \ if \ q_{i'}(k) = W_{i'}; \\ \alpha & : \ if \ q_{i'}(k) < W_{i'} \wedge q_i^{i'}(k) = 0; \\ (b_i^{i'}(k) - b_{i'}(k) + W^*)\mu_i^{i'} & : \ otherwise. \end{cases} \tag{8}$$

The parameters β and α are negative numbers, and we let

$$\beta < \alpha < 0. \tag{9}$$

With this, the gains in the two special scenarios discussed above are smaller than the case where at least some traffic flow is guaranteed through the junction if the link is activated. It is noted that β can also be larger than α, depending on the characteristics of the entire traffic network and preference of the traffic control authority.

For each control phase c_j, we define $g(c_j, k)$ as the sum of all constituent link gains,

$$g(c_j, k) = \sum_{L_i^{i'} \in c_j} g(L_i^{i'}, k). \tag{10}$$

We define $g_{\max}(c_j, k)$ as the maximum among all link gains,

$$g_{\max}(c_j, k) = \max_{L_i^{i'} \in c_j} g(L_i^{i'}, k). \tag{11}$$

B. The Proposed Algorithm

In this work, we propose a novel utilization-aware adaptive back-pressure traffic signal control algorithm as outlined in Algorithm 1. This algorithm is invoked at every time instant, thus enabling varying-length control phases. It takes as input the queue lengths $Q(k)$, the currently active control phase $c(k-1)$, and the global time t_k. It can be seen that all the inputs are local to the intersection. The algorithm decides whether to continue the current control phase, i.e., $c(k) = c(k-1)$, or to start the transition phase, i.e., $c(k) = c_0$. At the end of a transition phase, it decides a new control phase to apply $c(k) = c'$. Therefore, the output of the algorithm is the control phase at t_k, i.e., $c(k)$.

As explained below, the algorithm essentially considers three cases.

Case 1: It is currently a transition phase (c_0) and the transition period (Δk) has not expired (Line 1). In this case, no change is invoked (Line 2).

Case 2: It is currently not a transition phase and there exists a link in the current control phase with the gain higher than

Algorithm 1: The utilization-aware adaptive back-pressure traffic signal control algorithm

```
   Inputs           : Q(k), C, c(k-1), t_k
   Outputs          : c(k)
   Parameters       : W = {W_i | i ∈ NO}, Δk
   Global Variables: t_Δk
1  if c(k-1) == c_0 and t_k < t_Δk then
2  |  c(k) = c_0;
3  else if c(k-1) ≠ c_0 and g_max(c(k-1), k) > g*(k) then
4  |  c(k) = c(k-1);
5  else
6  |  if max_{c_j ∈ C} g_max(c_j, k) > α then
7  |  |  C' = {c_j | g_max(c_j, k) > α};
8  |  |  c' = arg max_{c_j ∈ C'} g(c_j, k);
9  |  else
10 |  |  c' = arg max_{c_j ∈ C} g_max(c_j, k);
11 |  end
12 |  if c' == c(k-1) or c(k-1) == c_0 then
13 |  |  c(k) = c';
14 |  else
15 |  |  c(k) = c_0;
16 |  |  t_Δk = t_k + Δk;
17 |  end
18 end
19 return c(k);
```

a non-negative threshold $g^*(k)$ (Line 3). The current control phase is then kept (Line 4). That is, we are happy with the current control phase, as long as it still offers a reasonably good junction utilization. This mechanism ensures that the control phases do not change too frequently and limits the number of transition phases. Note that $g^*(k)$ can be chosen based on customized requirements and traffic conditions. For example, we assume a value for $g^*(k)$ as follows:

$$\textbf{if}\ \ L_{\max}(c(k-1),k) == L_i^{i'}\ \ \textbf{then}\ \ g^*(k) = W^* \mu_i^{i'}. \tag{12}$$

In this case, our algorithm ensures that the queuing vehicles will be served until the pressure differences of all the constituent links become less than 0.

Furthermore, it can be seen that varying-length control phases are enabled. In the conventional back-pressure control, a control phase is applied for a pre-determined fixed-length time slot and each slot ends with a transition phase. In the proposed algorithm, we monitor the state of the intersection in every mini-slot of length $\Delta t = t_{k+1} - t_k$. One mini-slot is much smaller in size than a fixed-length slot in the conventional case. We stay in the same phase until a threshold is crossed. Therefore, each control phase can be applied for multiple mini-slots. The length of the control phase is thus variable and depends on several factors: (i) the state (queue lengths) of the intersection, (ii) the arrival of new vehicles to be served by the constituent links, (iii) the service rates of the links, and (iv) the capacities of the outgoing roads. Intuitively, a control phase can be extended if the junction utilization is reasonably good, and it can be cut short otherwise. This marks an important contribution of our work.

Case 3: Change of the control phase is invoked when the conditions of the above two cases are not satisfied. The algorithm first tries to find the control phase c' with the best link gain (Lines 6-11). This is an important feature of the proposed algorithm and makes it both utilization- and stability-aware. Here, it considers two scenarios. First, there exists one or more control phases which can guarantee some utilization of the junction in the next mini-slot (Line 6). In this scenario, the algorithm considers all such control phases and picks the one with the highest total gain, i.e., intuitively, the best effort against instability (Lines 7-8). Second, if it is not the first scenario (Line 9), the junction utilization will be low no matter which control phase is applied. The algorithm will pick a control phase with the highest link gain (Line 10). If the selected control phase c' is the same as the current one $c(k-1)$, then the traffic signals are not changed (Line 12-13). If it is currently the transition phase (expiring), then it will change to the selected c' (Lines 12-13). If the selected control phase c' is different from the current one $c(k-1)$ and the current one is not the transition phase (Line 14), then the transition phase must be activated (Line 15) and the expiry time of the transition phase is set (Line 16).

IV. PROPERTIES OF THE ALGORITHM

The back-pressure control algorithm guarantees maximum stability (in terms of bounded queue lengths) under idealistic assumptions. The modified back-pressure control method presented in [4] guarantees work conservation by its own (quite relaxed) definition, i.e., the junction works if there is at least one vehicle served during the fixed-length time slot. They do not consider the utilization of the junction. In this work, we propose to keep the essence of the back-pressure control and make it more utilization-aware. In this section, we analyze the properties of our proposed algorithm and answer some relevant questions in this regard as follows.

1. Does the algorithm guarantee maximum stability under all conditions? No. The maximum stability is guaranteed by the back-pressure control algorithm under idealistic assumptions. One such assumption is that the control phase change is immediate and can be triggered at any time, i.e., no transition phase is needed. Besides, the road capacities must be infinite. Our algorithm is developed towards the more realistic direction. In particular, it allows traffic flow even if the pressure difference between links is negative. This violates the stabilization principle where the pressure difference must tend towards zero. In addition, we consider roads with finite capacities as well as transition phases.

2. Is it possible to achieve work conservation with the algorithm? Yes, it guarantees work conservation. In queuing theory, an algorithm is work-conserving, if a server does not remain idle when there are customers to be served. It has been proved in [4] that the original back-pressure algorithm is not work-conserving and may result in deadlock and congestion propagation. The modified algorithm in [4] claims to achieve

TABLE I
TURNING PROBABILITIES OF VEHICLES ENTERING THE NETWORK

Entering from	North	East	South	West
Right-turning probability	0.4	0.3	0.4	0.3
Left-turning probability	0.2	0.3	0.3	0.4

TABLE II
AVERAGE INTER-ARRIVAL TIME OF VEHICLES ENTERING THE NETWORK

Pattern	Description	At each incoming road from			
		North	East	South	West
I	adjacent heavy	3 s	5 s	7 s	9 s
II	uniform	6 s	6 s	6 s	6 s
III	opposite heavy	3 s	7 s	5 s	9 s
IV	single heavy	3 s	9 s	9 s	9 s

work conservation itself, but only down to the fixed-length time slot.

In contrast, our algorithm guarantees work conservation in a much more strict sense, i.e., down to every time instant (or mini-slot). In particular, we let $g^*(k)$ be non-negative and thus larger than α. This means that before a control phase runs out of the vehicles to be served, a change of the control phase is considered. It is noted that for a control phase with only empty incoming queues and full outgoing queues, $g_{\max}(c(k-1), k) \leq \alpha$. On the other hand, the algorithm, while selecting a new control phase, always tries to pick one which has vehicles to be served (Lines 6-8 in Algorithm 1). The states of the intersection are monitored and the decisions on control phases can be made in every mini-slot. Therefore, our algorithm is work-conserving down to this time scale.

3. How does the algorithm improve the junction utilization? There are three points that contribute to the improvement of the junction utilization when compared to the conventional back-pressure control. (i) Our proposed algorithm enables varying-length control phases unlike the fixed-length control phases in the conventional case. Taking the worst case as an example, when all the outgoing roads of the current control phase are full, only one mini-slot is wasted at the maximum. When the control phases have fixed length, one slot, which is much larger than a mini-slot, could be wasted. (ii) In the original back-pressure control, the main objective is to stabilize the queue lengths between the incoming and the outgoing roads. Therefore, it does not allow traffic flow when the pressure difference is negative. Our method allows traffic flow when the pressure difference is negative. In many cases when there are more vehicles at the outgoing road than the incoming road (i.e., a negative pressure difference), the junction utilization can still be high. In addition, the mechanism keeping the current control phase instead of changing to the transition phase when the junction utilization is reasonably high, will limit the number of transition phases and thus improve the junction utilization. (iii) In the case of empty incoming roads, the gain is zero in the conventional back-pressure control. In the case of full outgoing roads, the gain can be zero [4] or a small value [3]. We differentiate these two cases from the general case with incoming roads that are not empty and outgoing roads that are not full. In the conventional back-pressure control, there is no such differentiation. For example, an equal number of vehicles at the incoming and outgoing roads results in the gain of zero. If the incoming road is not empty and the outgoing road is not full, the junction utilization will be high. If the incoming road is empty or the outgoing road is full, the junction utilization will be low or even zero. Treating these situations in the same way obviously compromises the junction utilization.

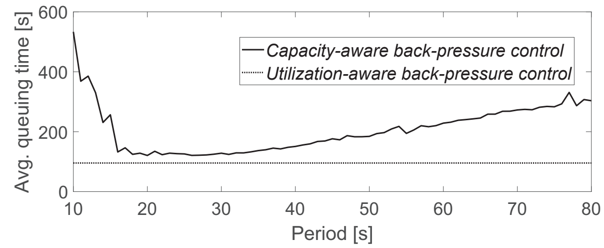


Fig. 2. Performance comparison for the mixed traffic pattern

TABLE III
COMPARISON RESULTS FOR ALL THE TRAFFIC PATTERNS

Pattern	*CAP-BP*		*UTIL-BP*
	Control Period	Avg. Queuing Time	Avg. Queuing Time
I	18 s	102.87 s	97.97 s
II	16 s	90.55 s	81.62 s
III	16 s	113.86 s	108.41 s
IV	22 s	125.63 s	94.05 s
Mixed	20 s	120.71 s	95.56 s

4. Is it possible to encounter head-of-line (HOL) blocking with the algorithm? In this work, we assume that vehicles going to different directions will queue up on different lanes. This is a realistic assumption in many metropolitan traffic networks at present. In such a setting, HOL blocking is not possible. However, it will be interesting to consider mixed lanes and devise an algorithm accordingly.

V. SIMULATION RESULTS

In the experiments, we set up a 3×3 traffic network with 9 traffic intersections, where each intersection is as described in Section II-A and shown in Figure 1. Turning probabilities of vehicles entering the network from different directions are given in Table I, while the intersection at which a vehicle takes the turn is selected randomly. The duration of the transition phase is $4\,\mathrm{s}$ when the amber light is on. The capacity of each road is assumed as $W_i = 120$, where $i \in \mathcal{NO}$. We assume $\alpha = -1$ and $\beta = -2$. We define $g^*(k)$ as given in (12). For each feasible link $L_i^{i'}$, we assume a maximum service rate $\mu_i^{i'} = 1$. Four different traffic patterns are considered, where the average inter-arrival time of vehicles entering the network at different incoming roads are given in Table II. For instance, in Pattern I, at each incoming road in the north of the network, vehicles will enter every $3\,\mathrm{s}$ on average. Each traffic pattern is simulated in the widely used microscopic simulator SUMO [7] for $1\,\mathrm{h}$. In addition, we consider a mixed pattern of $4\,\mathrm{h}$ combining the four traffic patterns.

We simulate both the fixed-length algorithm [4], hereafter named *CAP-BP* as it is called capacity-aware in that paper, and the proposed utilization-aware back-pressure control, hereafter

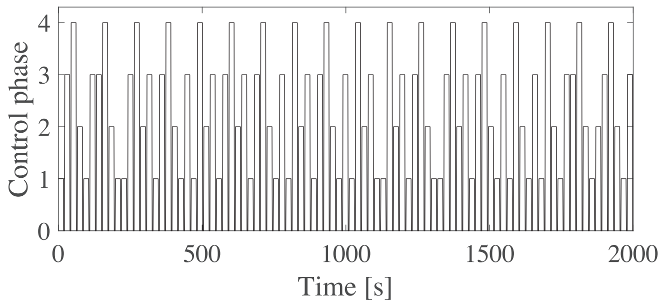


Fig. 3. Applied control phases on the top-right intersection using the fixed-length capacity-aware back-pressure control for Pattern I (considering the optimal period)

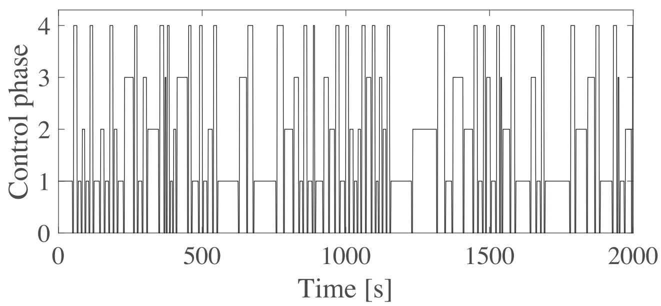


Fig. 4. Applied control phases on the top-right intersection using the proposed utilization-aware back-pressure control for Pattern I

named *UTIL-BP*. In Figure 2, the solid line shows the variation of the average queuing time of a vehicle (in the entire network) with the value of the control phase period when *CAP-BP* is used for the mixed traffic pattern. Here, the control phase period is set globally for the traffic network under study. That is, every intersection is controlled using the same period. Note that even the best possible average queuing time of a vehicle that can be achieved using *CAP-BP* is longer than the time obtained using *UTIL-BP*. In Table III, we report the comparison results for all the patterns. In the case of *CAP-BP*, only the best possible average queuing time is shown. On average, *UTIL-BP* performs 13 % better than the best possible results obtained with *CAP-BP*. This is a significant improvement in the context of traffic signal control. Moreover, it may be observed in Table III that the optimal value of the control phase period for *CAP-BP* depends on the traffic pattern. Thus, choosing this value would require prior knowledge of the traffic which might not be possible, and thus undermine a major advantage of the back-pressure algorithm.

For better visualization, we consider Pattern I for 2000 s as an example, and plot the applied control phases on the north-eastern (top-right) intersection using the two algorithms in Figure 3 and Figure 4, respectively. With heavier traffic coming from the north/south and more vehicles going straight/turning left, the *UTIL-BP* algorithm demonstrates better adaptability, where longer phase periods are assigned to the control phase 1 and 2. The Poisson distribution of the traffic patterns also contributes to the accumulation of vehicles in certain periods. In contrast, *CAP-BP* (considering the optimal period) maintains fixed phase length and is less flexible to unbalanced traffic patterns, and hence, resulting in longer average queuing time. The queue lengths for the two algorithms at the incoming road from the east at the same intersection (top-right) is shown in Figure 5. *UTIL-BP* has a shorter queue length than *CAP-BP* in general.

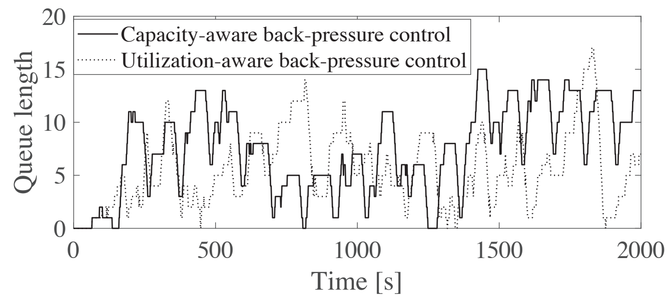


Fig. 5. Queue lengths at the incoming road from the east on the top-right intersection for the two control algorithms

VI. Concluding Remarks

In this paper, we propose a novel utilization-aware adaptive back-pressure traffic control algorithm. It takes both the stability and utilization of an intersection into account while making control decisions. Traffic flow is allowed through the junction even when the pressure difference is negative, in order to improve the utilization. The proposed algorithm considers finite road capacities acting as upper bounds to queue lengths and empty roads. Varying-length control phases allow the decision to be changed when low utilization is noticed. In future, we would like to simulate a real-world network of intersections using this algorithm and evaluate its performance. Furthermore, it is interesting to study mathematically the trade-off between stabilization and utilization. As we consider more realistic setting of a signalized intersection, proving relevant properties of this algorithm can be pursued.

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