Abstract—Memristor based logic and memories are increasingly becoming one of the fundamental building blocks for future system design. Hence, it is important to explore various methodologies for implementing these blocks. In this paper, we present a novel Complementary Resistive Switching (CRS) based stateful logic operations using material implication. The proposed solution benefits from exponential reduction in sneak path current in crossbar implemented logic. We validated the effectiveness of our solution through SPICE simulations on a number of logic circuits. It has been shown that only 4 steps are required for implementing N input NAND gate whereas memristor based stateful logic needs N+1 steps.

I. INTRODUCTION

Implication based stateful logic using memristors has recently been proposed as a design paradigm that allows to implement any boolean logic using memristors. Further, memristor based resistive-variable memory devices have recently emerged as a promising technology to overcome the limitations of traditional CMOS based memories [1] [2] [3]. These devices can be fabricated with finer layouts using non-lithographic methods like imprint lithography to produce high density memory with short access latencies. Moreover, they exhibit non-volatile state retention characteristics, which are particularly suitable for stable data storage. Hence, memristor based efficient memory design and implementation is currently of high interest both in academia and industry. Memristive devices can also be programmed for several different resistance states that features a multilevel RRAM [4].

Over the years various memristor array based memory architectures have been reported. For example, passive crossbar arrays of memrisitve elements were reported as possible non-volatile random access memories (RAMs) in [5]. However, passive crossbar arrays have the general issue with sneak-path currents due to interference from the neighbouring cells when selecting a designated cell within the arrays. To avoid sneak-path currents, recently complementary resistive switches (CRS) were proposed [2], which consist of two anti-serial memristive elements. These elements, including a high resistance state and a low resistance state, are used to encode either logic 0, 1, ON or OFF (See Fig 1). The transitions between these states depend on the applied biasing voltage across the CRS, controlled by SET and RESET threshold voltages [2].

Stateful logic operation based on material implication (IMP) was first proposed by HP Laboratories [7]. The basic structure of the material implication proposed by HP Laboratories is discussed in the next section. There are many proposals that can intrinsically enable logic in memory architectures, for which the non-volatile memory elements are used as the main devices for logic computations [7]. In [6] authors proposed a circuit structure that performed logic operation on memristor memory based on nanocrossbar. Here the authors assumed basic memristor module, however in such a large array sneak path could be a problem.

In this paper we propose a CRS based stateful logic operations using material implication. To the best of our knowledge, no prior work exists implementing CRS based stateful implication logic. The proposed solution benefits from exponential reduction in sneak path current in crossbar implemented logic. Further, it has been shown that only 4 steps are required for implementing N input NAND gate whereas memristor based stateful logic needs N+1 steps. The rest of the paper is organized as follows. Section II introduces preliminary concepts related to this paper, Section III describes the proposed CRS stateful logic and NAND operation. Experimental results and crossbar sneak path analysis are presented in Section IV. Finally, Section V concludes this paper.

II. BACKGROUND AND MOTIVATION

A. Memristor IMP Operation

Memristor is a two terminal device which has two states, 1 for low resistance state ($R_{ON}$) and 0 for high resistance state ($R_{OFF}$) [1], [7]. The transitions between these states depend on the applied biasing voltage across the memristor, controlled by SET and RESET threshold voltages. In [7], it is shown that memristor can be used for stateful logic operation. The memristor IMP operation is shown in Fig. 2. As Fig. 2(a) shown, two memristors are connected with a nanowire (the selected line). One terminal of those memristors will be connected together with a load resistor $R_L$. Voltages $V_{COND}$ will be applied across P memristor and $V_{SET}$ will be applied to Q memristor. After that, the state of Q memristor is the IMP operation result. The truth table for memristor IMP operation is shown in Fig. 2(b). To perform the IMP operation, the following Eqn. $R_{ON} < R_L < R_{OFF}$ needs to be satisfied [7]. $V_{SET}$ is the positive voltage which can shift memristor to state 1. $V_{COND}$ is the positive voltage less than $V_{SET}$, thus cannot change the state of memristor. When P memristor is in high resistance state 0, most of $V_{COND}$ is dropped across P memristor. Therefore voltage across Q memristor is equal to $V_{SET}$, the state of memristor Q changes or stays on state 1. On the other hand, if memristor P is in state 1, the voltage on selected line is equal to $V_{COND}$, the voltage across memristor Q is not enough to switch, it will remain its state 0.

![Fig. 2. (a)IMP operation of memristor cell. (b)Truth table of IMP](image-url)
B. Memristor Memory Logic

Fig. 3(a) shows a stateful logic mapped to crossbar structure, voltage \( V_{SET} \) and \( V_{COND} \) should be applied to the memristor \( P \) and \( Q \) in logic operation stage. Logic will be performed between those two memristors and output will be stored in memristor \( Q \). \( R_G \) is used as a pull-down resistor in logic operation stage. To read the output result in crossbar structure, a read scheme [8] is proposed. A voltage \( V_{READ} \) is applied to the reading row, \( R_G \); in read stage will act as a load resistor, voltage on \( R_G \) will be sensed and compared with \( V_{REF} \). One of problems in this scheme is sneak path current that exists during the stateful logic operation. In Fig. 3(a), the red line with arrow shows the sneak path current. This sneak current can pass the non-operational memristors and inject the current to the selected line pulling up the voltage and this could corrupt the output results. For example, if the two red circled operation cells are in high resistance state. Apply \( V_{SET} \) to \( Q \) memristor can switch it to low (low resistance state). However, due to sneak path current, the voltage in selected line will be pulled up, so the voltage for \( Q \) memristor will be much less than \( V_{SET} \). As a result, memristor \( Q \) remains in its state, and an error is induced. To address this issue, a design using CRS is presented in [2] to reduce the sneak path current.

III. CRS STATEFUL LOGIC

A. CRS IMP Operation

As explained in Section I, CRS has four different states as shown in Fig. 1. Note that state 0 and state 1 have the same resistance state \( (R_{CRS}) \) as one of memristors \( X \) and \( Y \) is in \( R_{OFF} \) and the other is in \( R_{ON} \) state. When both of those memristors are in \( R_{ON} \) states, the CRS is in state ON (the lowest resistance). When both of them are in \( R_{OFF} \) state, CRS is in OFF state and this state is not used in stateful logic.

To perform CRS IMP operation, consider the Fig. 4. Fig. 4(a) shows the basic structure. This structure contains two CRS cells \( CRS_P \) and \( CRS_Q \), result will be latched in \( CRS_Q. \) By applying voltage \( V_P \), \( V_Q \) to \( CRS_P \) and \( CRS_Q \) respectively, the circuit can perform IMP operation \( CRS_Q^{\prime} = CRS_P \) IMP \( CRS_Q. \). Fig. 4(b) shows the truth table of CRS IMP. Three states 1, ON, 0 are used in CRS IMP. Fig. 4(c) shows the number of steps to perform the CRS IMP operations and applied voltages for each step. During IMP operation, \( V_{ONCRS-1} \), \( V_{CONDCRS-1} \) and \( V_{CLEARP} \) (these are defined in Fig 5) are applied to \( CRS_P \), and \( V_{SETQ} \) is applied to \( CRS_Q. \) In memristor stateful logic, if the applied voltage is not sufficient to switch the state to its opposite state, memristor will remain in its current state. However in CRS IMP operation, the state of CRS may be changed to state ON. During stateful logic operation,
CRS_Q from state ON. Also, if R_G is smaller than CRS_P state ON (R_CRSONP), V_COND CRS-1 will be blocked by CRS_P and the voltage at the selected line will be low. To solve those issues, select R_G < R_CRSONQ and greater than R_CRSONP. The variation of R_G will effect the successful CRS IMP operation. Increasing the value of R_CRSONQ and decreasing R_CRSONP can make the circuit has a high tolerance of the variation. In equation (7), R_CRS is the resistance value for state 0 and 1 of one CRS.

\[ V_{PH1} > V_{QH1}, V_{PH2} > V_{QH2} \]  \hspace{1cm} (1)

\[ V_{PH3} < V_{QH4}, V_{PH4} < V_{QH4} \]  \hspace{1cm} (2)

\[ V_{ONCRS-1} < V_{PH3} \]  \hspace{1cm} (3)

\[ V_{QH2} - V_{QH1} < V_{COND CRS-1} < V_{PH1} \]  \hspace{1cm} (4)

\[ V_{CLEARP} < V_{PH4} \]  \hspace{1cm} (5)

\[ V_{SETQ} > V_{QH2} \]  \hspace{1cm} (6)

\[ R_{CRSONP} < R_G < R_{CRSONQ} << R_{CRS} \]  \hspace{1cm} (7)

Fig. 4(c) shows the steps for CRS IMP operation. First, CRS_P and CRS_Q are configured to their initial states. Then in Step -1, only voltage V_{ONCRS-1} is applied to CRS_P to shift it to state ON if it is in state 1. The operation in Step - 2 is like memristor IMP, voltages V_{COND CRS-1} and V_{SETQ} are applied simultaneously, if CRS_P is in state ON, the voltage on selected line will be pulled to V_{ONCRS-1}, thus for CRS_Q, the voltage dropped across it is less than V_{SETQ} and the CRS cell keeps stable, otherwise, if CRS_P is in state 0, it blocks V_P, voltage across CRS_Q equal to V_{SETQ}, CRS_Q will be shifted to state 1. After operation steps, CRS_P may come into state ON (low resistance state). So it induces sneak path current into crossbar structure. To address this issue, one extra reset step Step - 3 is performed after operation steps. In this step, CRS_P will be reset to state 0. The CRS crossbar logic is shown in Fig. 3(b).

From this figure, memristors are replaced by CRS cells and CRS_Q are always in the reading row.

B. CRS NAND Operation

According to [7], any logic can be implemented using NAND gates. So the following section will illustrate the details of NAND CRS operations. The two input NAND operation needs three CRS cells, CRS_P1, CRS_P2 (NAND inputs) and CRS_Q which will get the result after NAND operation. To perform NAND operation, CRS_Q is set to state 1 under voltage V_{SETQ} in Step -1 (see Fig. 6(b)). In Step-2, V_{ONCRS-0} is applied to CRS_P1 and CRS_P2 to switch them to state ON if they are in state 0. Step - 3 is the most important part of NAND operation, voltage V_{COND CRS-0} and V_{CLEARQ} are applied to all operation CRS cells simultaneously. If both CRS_P1 and CRS_P2 are in state 1, CRS_Q will be shifted to state 0. Otherwise, CRS_Q will stay in state 1, because most of voltage V_{COND CRS-0} is dropped across R_G and make CRS_Q has no enough switch voltage. From Fig. 6(c), it can be seen that if either CRS_P1 or CRS_P2 is in state 0, CRS_Q remains in state 1, otherwise it is changed to state 0 which matches the NAND truth table. Finally, one reset step for CRS_P1 and CRS_P2 is required (See Fig. 6(c)).

To perform N inputs CRS NAND operation, it only needs four steps, however N + 1 CRS cells are required. CRS_Q will be initialized to state 1 in Step - 1, then all N input CRS_P cells will be performed NAND operation simultaneously under voltage V_{ONCRS-0} and result will be latched in CRS_Q. Further, one reset step is followed. For memristor NAND operation, in previous approach, it needs to perform N times IMP, so N inputs memristor NAND logic requires N + 1 steps (N IMP steps plus one step initialization). The number of steps required for multi input NAND operation is shown in Table I.

### TABLE I

<table>
<thead>
<tr>
<th>N inputs NAND Operation using IMPLY</th>
<th>CRS Proposed Method</th>
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<tbody>
<tr>
<td>IMP</td>
<td>3 &lt; 1</td>
</tr>
<tr>
<td>2 inputs NAND</td>
<td>4 - 3</td>
</tr>
<tr>
<td>3 inputs NAND</td>
<td>4 - 4</td>
</tr>
<tr>
<td>4 inputs NAND</td>
<td>4 - 5</td>
</tr>
<tr>
<td>N inputs NAND</td>
<td>4 - N+1</td>
</tr>
</tbody>
</table>

IV. Simulation Results and Analysis

A. Simulation Results

The following explains the simulation results performed for various stateful logic in CRS using Cadence-Spectre simulation tool. To perform the CRS stateful logic operation, the parameters of CRS cells and R_G should follow the equations (1)-(7). Here, we choose R_{CRSONP} = 200Ω for state ON of CRS_P and R_{CRSONQ} = 2Ω for state ON of CRS_Q. The value of R_G should be larger than R_{CRSONP} and R_{CRSONQ}. R_G = 1kΩ. Adding R_G will increase the V_{PH2} and V_{QH2} [9], we used 5.3V for V_{SETQ}, −5.6V for V_{CLEARQ}. According to apply voltages definition in equations (3), (4) and (6), V_{ONCRS-1} is given the value −5.5V, V_{COND CRS-0} is 5.5V. V_{COND CRS-1} = 3.6V, V_{COND CRS-0} = −4.2V. The reset voltage V_{CLEARQ} = −5.7V.

Fig. 7 shows the CRS IMP and NAND simulation results. Fig. 7(a), (c) are the applied voltages for CRS IMP and NAND operation. In Fig. 7(b), (d), the state of CRS_Q and CRS_P is represented by two memristors called state QX, state PX (the top red memristor) and state QY, state PY. (the bottom black dashed memristor) respectively. The x-axes in Fig. 7(b), (d) represent the doping length of the memristor in nanometer (nm). When the doping length is 3nm, memristor is in state 1 and when the doping length is 0nm, memristor is in state 0 [11]. So if CRS_Q is in state 1, state QX, the top memristor should be 3nm, the bottom memristor state QY should be 3nm (See Fig. 7(b)). Comparison between Fig. 7, Fig. 4 and Fig. 6 proves that the simulation results are matched with the expected output discussed in Section III.

B. Sneak Path Analysis

In this section, we derive sneak path voltage equations for memristor followed by CRS. Here we assume the worst case for the crossbar logic operation. The selected operating memristors P and Q in crossbar array are in state 0 (high resistance), all other memristors are in state 1 (low resistance), and voltage V_{COND}
the ratio of $R$ in crossbar array, x-axis is the crossbar array size. From this figure, effectively. However when $R$ calculation, memristor crossbar logic has a worse sneak path current $R_{\text{sneak}}$ effect is reduced. Due to $V_{\text{path}}$ calculation. When the ratio of $R$ for CRS. The result of $V_{\text{c}}$ is expressed in (8).

$$V_{\text{Msneak}} = \frac{V_{\text{COND}} + R_{G}}{m+1} \times R_{ON}$$ (8)

For CRS based crossbar logic, every CRS cell is in state 1 or state 0. So they have the same resistance values $R_{\text{state}} = R_{\text{state1}} \approx R_{\text{CRS}}$. Assume the cells in crossbar first row are $CRS_P$ and $CRS_S$ has $m \times m$ matrix as Fig. 3(b) shows. Both operation CRS cells are in state 0. Other cells are in state 1. The sneak path voltage $V_{\text{Cmsneak}}$ affected by sneak path current in CRS crossbar is shown in equation (9).

$$V_{\text{Cmsneak}} = \frac{V_{\text{COND}} + R_{G}}{m+1} \times R_{CRS}$$ (9)

Fig. 8. Sneak path worst case comparison

Fig. 8 shows the worst case sneak path comparison between memristor and CRS. In this figure, y-axis is $V_S/V_C$ in log scale. $V_S$ represents $V_{\text{Msneak}}$ for memristor crossbar array and $V_{\text{Cmsneak}}$ for CRS. $V_C$ is $V_{\text{COND}}$ for memristor crossbar and $V_{\text{CONDcrs}}-1$ for CRS. The result of $V_S/V_C$ expresses the impact of sneak path in crossbar array, x-axis is the crossbar array size. From this figure, the ratio of $RG$ with $R_{ON}$ and $R_{CRS}$ plays a key role in sneak path voltage calculation. When the ratio of $V_S/V_C$ gets smaller, the sneak path effect is reduced. Due to $R_{ON}$ for memristor worst case calculation, memristor crossbar logic has a worse sneak path current issue. Increasing the value of $R_{ON}$ can reduce the sneak path current effectively. However when $R_{ON}$ is equal to or larger than $RG$, this can make the memristor stateful logic operation unsuccessful. For CRS crossbar logic, $R_{CRS}$ is much larger than $RG$ which makes the sneak path effect minor. Increasing the value of $R_{CRS}$ can improve the performance.

V. CONCLUSIONS

CRS can effectively reduce the sneak path in CRS arrays like crossbar structure. CRS based crossbar is more suitable than memristor based arrays for stateful logic operation. In this paper, a CRS stateful logic scheme is explored. IMP operation can be performed with two different types of CRS cells. We have demonstrated CRS IMP and NAND operation. One additional step is required compared to memristor IMP operation. Using basic NAND operation, all logic function can be implemented. Furthermore, only 4 steps are required for implementing N input NAND gate whereas memristor based stateful logic needs N+1 steps. The most important feature of our proposed NAND CRS logic is its reduction in sneak path current and constant computational steps, which will be very useful for logic design compared to memristor implication which requires variable number of steps depending on the number of inputs.

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